MEASUREMENTS, UNITS AND DIMENSIONS

1. Fundamental Physical quantites:
The physical quantity which does not depend upon other physical quantities.

2. Derived Physical quantities:
The physical quantities which can be derived from the fundamental quantities are called as derived quantities.

3. System of units.

(a) **The f.p.s system** is the British Engineering system of units, which uses foot as the unit of length pound+ as the unit of mass and second as the unit time.

(b) **The c.g.s system** is the Gauusian system which uses centimeter, gram and second as the three basic units for length, mass and time respectively.

(c) **The m.k.s system** is based on metre, kilogram and second as the fundamental units of length, mass and time respectively.

(d) **International system of units (SI)**

4. Advantages of SI
1. SI is a coherent system of units
2. SI is a rational system of units
3. SI is an absolute system of units
4. SI is a metric system

5. Dimensional formula:
The expression showing the relation between the derived quantity and fundamental quantities by raising the powers of the fundamental quantities is known as dimensional formula.

6. Dimensions:
The dimensions of a physical qunatities are the powers up to which the fundamental quantities are to be raised.

7. Principle of homogeneity,
In any dimensional equation if dimensions of L.H.S terms are same to that of dimensions of R.H.S terms, that is known as principle of homogeneity.

8. Uses of dimentional formula
a. To check the validity of the given equation.

b. To change units of a physical quantity from one system into the other.

c. To derive relationship between the different physical quantities.

9. Limitation of Dimensional Analysis

a. This method gives us no information about the dimensionless constants in the formula, e.g. 1,2,3….π, e, etc.

b. If a quanity depends on more than three factors, having dimensions, the formal cannot be derived. This is because on equating the powers of M,L and T on either side of the dimensional equation, we can obtain three equation, from which only three unknown dimensions can be calculated.

c. We cannot derive the formulae containing trigonometrical functions, exponential functions, log functions etc, which have no dimensions.

d. It gives no information whether a physical quantity is a scalar or vector.

10. Always in a physical quantity the numerial value is inversly proportional to its units.

$$N \propto \frac{1}{U}$$

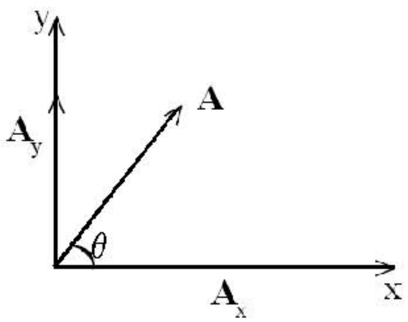
$$N_1 U_1 = N_2 U_2$$

VECTORS

1. The physical quantities having both magnitude and direction and also obeying laws of vectors are known as vectors.

2. **Different types of vectors**
   a. **Parallel vectors:**
   If two or more vectors are parallel to another, they are said to be parallel vectors.
   b. **Equal vectors:**
   If two or more vectors have equal magnitude and acting in the same direction, they are said to be equal vectors.
   c. **Negative vector:**
   If two vectors $\vec{A}$ and $\vec{B}$ are such that they have equal magnitude but opposite directions, each vector is negative of the other
   d. **Null vector:**
   A vector of zero magnitude with indefinite direction.
   e. **Unit vector:**
   A vector whose origin and terminous lie at same point.
   f. **Position vector:**
   The vector used to specify the position of a point with respect to some fixed point (say origin 'O') is called position vector.

3. **Resolution of Vectors**
   Definition: The process of dividing a vector into its components is called resolution of vector.
   a. Vector into two components



Horizontal component $A_x = A\cos\theta$

Vertical component $A_y = A\sin\theta$

Always $A = \sqrt{A_x^{\,2} + A_y^{\,2}}$

b. Vector into three components

$A_x = A\cos\alpha, A_y = A\cos\beta, A_z = A\cos\gamma$

Direction of cosines

$$Cos\alpha = \frac{A_x}{A};\ Cos\beta = \frac{A_y}{A};\ Cos\gamma = \frac{A_z}{A}$$

Where α, β and γ are the angles made by $\vec{A}$ with x, y and z-axes respectively

4. **Parallelogram law of vectors**
   Statement:
   If two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, the diagonal passing through that point represents their resultant both in magnitude and direction.

   Formula: $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$;

$$Tan\theta = \frac{B\sin\theta}{A + B\cos\theta}$$

5. **Trianlge law of vector**
   Statement:
   If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, the third side of the triangle taken in reverse order represents their resultant in mangitude and direction.

Components of vector

6. Vector $\vec{A}$ can be represented as sum of three vectors $\vec{A_x}\vec{A_y}$ *and* $\vec{A_z}$ along the three specified axes x, y and z Thus

$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

Vector $\vec{A_x}, \vec{A_y}$ *and* $\vec{A_z}$ are know as components of vectors. At this stage we Define unit vector as a vector having a magnitude of unity with no unit Its only Purpose is describe a direction in space with the help of three unit vectors

$\hat{i}, \hat{j}$ *and* $\hat{k}$, Respectively along x, y and z axis, we can write:

$\overrightarrow{A_x} = A_x\hat{i}, \overrightarrow{A_y}\hat{j}, \overrightarrow{A_z} = A_z\hat{k}$

and $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

giving the modulus of vector $\vec{A}$ as

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Also if α, β and γ are angles of $\vec{A}$ with x, y and z-axis, intern, then

$$I = \cos\ \alpha = \frac{A_x}{A}, m = \cos\beta = \frac{A_y}{A},$$

$$n = \cos\gamma = \frac{A_z}{A}$$

Where I, m and n we define as direction cosines of vector giving the relation

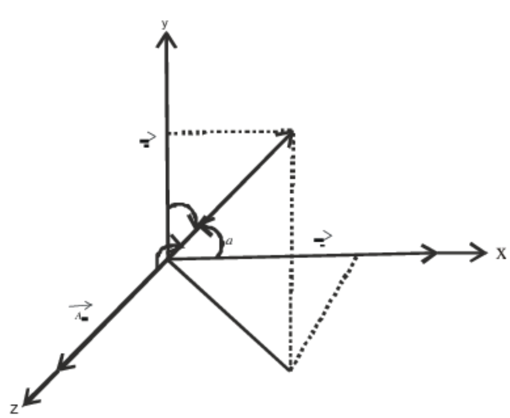
$$I^2 + m^2 + n^2 = 1$$

Resolution of vector in a plane may be expressed as $\vec{A} = A_x\hat{i} + A_y\hat{j}$

Also if vector $\vec{A}$ makes an angle θ with x-axis then

$A_x = A\cos\theta$ *and* $A_y = A\sin\theta$ giving the amplitude

$$A = \sqrt{A_x^2 + A_y^2}$$



7. Relative velocity (man walking in rain)

Formula: $\tan\theta = \frac{V_m}{V_r}$

where

V_m = Velocity of man

V_r = Velocity of rain

8. Motion of a boat in a river in shortest path (Up the stream)

Formulae:

a. $Sin\theta = \frac{V_w}{V_b}$

where

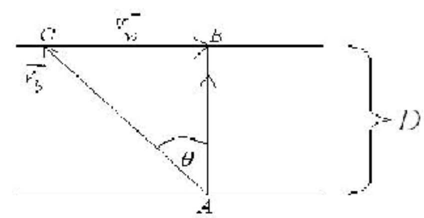
V_w = velocity of water

V_b = velocity of boat

b. V_R = Resultant velocity = $\sqrt{V_b^2 - V_w^2}$

c. time taken to cross the river

$$t = \frac{D}{\sqrt{V_b^2 - V_w^2}}$$

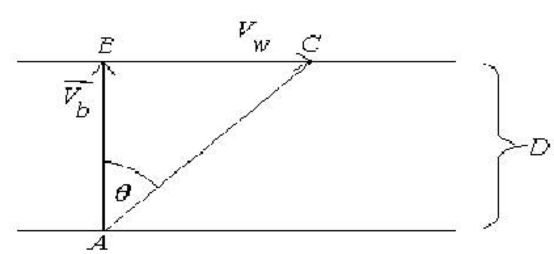


9. Motion of a boat in a river in shortest time (Down the stream)

Formulae:

a. time (minimum) = $t = \frac{D}{V_b}$

b. BC = Drift = $D\left(\frac{V_w}{V_b}\right)$



10. Dot Product

Definition of dot product:

The dot product of two non zero vectors is defined as the product of their magnitude and cosine of the angle between them.

Formulae: $\vec{A} \cdot \vec{B} = AB\cos\theta$

Dot product of two vectors always yield a scalar.

Dot product obeys

a. commutative law **a.b = b.a**

b. Distributive law **a.(b + c) = a.b + a.c**

c. Dot product can be -Ve, +Ve, and 0

d. Dot product of unit vectors

i.i = j.j = k.k = 1

i.j = j.k = k.i = 0

11. Vector Product

The vector product of two non-zero vectors is defined as the product of their magnitude and angle of sign between them.

Formula: $\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$ where $\hat{n}$ is the unit vector perpendicular to plane containing $\vec{A} \times \vec{B}$.

the vector product of two vectors is always a vector.

The directions of vector product always lie perpendicular to the plain containing two vectors in accordance with right handed system.

Properties:

a. does not obeys commutative law

$$\mathbf{a}\times\mathbf{b} \neq \mathbf{b}\times\mathbf{a}$$

b. Obeys distributive law

$$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$$

c. Cross product between unit vectors

$$\mathbf{i}\times\mathbf{j} = \mathbf{k}$$

$$\mathbf{j}\times\mathbf{k} = \mathbf{i}$$

$$\mathbf{k}\times\mathbf{i} = \mathbf{j}$$

If $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

$$\therefore \quad \vec{a}\times\vec{b} = \hat{i}\left(a_y b_z - b_y a_z\right) - \hat{j}\left(a_x b_z - b_x a_z\right) + \hat{k}\left(a_x b_y - b_x a_y\right)$$